Efficient and Distributed SINR-based Joint Resource Allocation and Base Station Assignment in Wireless CDMA Networks

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Abstract

We formulate the resource allocation problem for the uplink of code division multiple access (CDMA) networks using a game theoretic framework, propose an efficient and distributed algorithm for a joint rate and power allocation, and show that the proposed algorithm converges to the unique Nash equilibrium (NE) of the game. Our choice for the utility function enables each user to adapt its transmit power and throughput to its channel. Due to users' selfish behavior, the output of the game (its NE) may not be a desirable one. To avoid such cases, we use pricing to control each user's behavior, and analytically show that similar to the no-pricing case, our pricing-based algorithm converges to the unique NE of the game, at which, each user achieves its target signal-to-interference-plus-noise ratio (SINR). We also extend our distributed resource allocation scheme to multi-cell environments for base station assignment. Simulation results confirm that our algorithm is computationally efficient and its signalling overhead is low. In particular, we will show that in addition to its ability to attain the required QoS of users, our scheme achieves better fairness in allocating resources and can significantly reduce transmit power as compared to existing schemes.

Index Terms

Distributed joint resource management, game theory, Nash equilibrium, pricing.

I. Introduction

As the demand for high speed services with different quality of service (QoS) requirements increases, it is becoming more desirable to improve the efficiency of using the scarce radio resources. However,

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temporal variations of system parameters, such as channels' gains, make this a difficult task. Radio resource allocation in wireless networks exploits temporal variations in different users to allocate available resources in such a way that under some system and service constraints, a performance measure for each user, e.g., the total throughput or the total transmit power, is optimized.

In distributed resource allocation, each user determines the amount of resources to be utilized by that user so that a measure of QoS can be satisfied. Game theory [1] is an efficient tool that is commonly used in the analysis of distributed resource allocation schemes. In this context, the problem of resource allocation is formulated as a non-cooperative game in which each user chooses a strategy from its strategy space, such that its utility function is maximized. Distributed power control has been widely studied in the literature [2]–[5]. In power control, each user chooses a power level such that its utility function is maximized in terms of a QoS measure. The utility in [3] is channel capacity, in [4] is energy efficiency, and in [5] is a sigmoidal function that approximates the probability of successful transmission.

Although power control aims to efficiently allocate power levels to users, if available resources (e.g., power levels, data rates, and base stations) are jointly allocated, it may be possible to improve the system's performance even further. However, this comes at a price. Joint allocation of resources results in multi-dimensional strategy spaces for users, and requires more complex utilities, which makes the analysis of the corresponding game more difficult. Besides, the amount of information needed in distributed algorithms by each user for joint allocation of resources is much more than those of single resource allocation algorithms. Moreover, efficiency of algorithms and their convergence in joint allocation of resources depend on the utility function and pricing.

Joint resource allocation has already been addressed in [6]–[15]. Joint power allocation and base station assignment is considered in [6], [7]. In [6], the minimum power required to achieve a target signal-to-interference-plus-noise ratio (SINR) is obtained; and base stations are assigned in such a way to maintain a minimum SINR for each user. This approach is satisfactory for voice service. However, for data services, a more efficient algorithm is desirable. In [7] the power control framework proposed in [4] is extended for a multi-cell scenario by modeling the resource allocation problem as a non-cooperative game in which each user maximizes its own utility, and the performance of the algorithm depends on the value of pricing.

In [8], [9], joint rate and power allocation for the downlink of power limited CDMA networks is studied. The authors in [8] show that it is optimal for each base station to transmit to not more than one data-user at any given time. This scheme is unfair, since users with bad channels or those that are located farther from the base station would not be allowed to transmit, as the algorithm only allows the user with

the best channel to transmit. In [9], the authors use scheduling together with power control to manage interference. In [10], joint data rate and power control for CDMA uplink is obtained by solving the corresponding optimization problem via the dual Lagrange approach, which needs excessive calculations and high feedback overhead. In [11], joint data rate and power control is considered by utilizing a layered game theoretic approach consisting of two games. In the first game, each user chooses a rate form its rate strategy space to maximize its utility function. When this game converges, the target data rate of each user is determined, and the second game starts. In the second game, each user chooses a power level to maximize its utility function. As such, each user plays two consecutive games. In the first game, there are some parameters that need to be determined in such a way that the user's data rate is efficient and achievable. However, [11] is silent on how these parameters are set. Besides, at least the sum of the rates of all users should be feedbacked in the first game; and the total received power should also be feedbacked by the receiver in the second game.

In [12], [13], a joint data rate and power control scheme that uses the utility proposed in [4] and QoS is a function of data rate and queuing delay is considered. The effects of modulation and constellation size on joint data rate and power control scheme in [12] is considered in [13], where the chosen utility function makes it possible to have many optimal data rates and power levels, i.e., multiple Nash equilibria (NEs) exist. In [14], the same utility function as in [12], [13] is used and a joint data rate and power control scheme that may have multiple NEs is proposed, but no analysis on NEs is provided.

In [15], joint data rate, power control, and base station assignment for the downlink of CDMA systems is considered with a view to maximizing the sum of all users' utilities under total power constraint, where the optimal data rate is obtained in terms of the allocated transmit power, i.e., the problem is converted into a power control one. The optimization problem is solved in two steps: user selection, and power allocation; and it is shown that its solution approaches an optimal one when the number of users is high. However, it utilizes extensive iterative signalling, and its base station assignment is sub-optimal.

In this paper, we use game theory [1] to formulate the joint resource allocation problem for the CDMA uplink. Each user tries to maximize its utility over the data rate and power level strategy space, independent of other users. Our choice for the utility function enables each user to efficiently and adaptively choose its transmit power and data rate. In other words, to maintain an acceptable QoS, each user increases its transmit power and/or decreases its throughput when its channel is not good, and increases its throughput and decreases its transmit power when its channel is good. In this way, users with bad channels cause less interference, resulting in higher throughput values for users with good channels, and in more users supported by the network.

Due to users' selfish behavior, the output of the game (its NE) may not be a desirable one, meaning that in such cases, users transmit at their highest power levels and data rates without achieving their QoS. To avoid such cases, we use pricing to control the selfish behavior of users, and propose a distributed algorithm for joint data rate and transmit power control. We will show that the proposed game with pricing has a unique NE at which no user can improve its utility by unilaterally changing its strategy, and prove that our algorithm converges to the unique NE of the game.

We further demonstrate that in addition to its distributed nature, the overhead in our approach is low as compared to other existing approaches. In particular, we will show that our algorithm needs the same information as the target SINR-tracking power control algorithm (TPC) [2] that is known to have a very low overhead. Note that the strategy space of users in our approach is two dimensional, which makes the problem more complex as compared to that of TPC. Nevertheless, we will also show that the proposed scheme is as computationally efficient as TPC. Moreover, we will show that in our proposed scheme, each user will achieve its predefined SINR level at NE. In addition, we will extend our proposed scheme to multi-cell networks by assigning base stations to users, and show that for the joint data rate and power level and base station assignment, there exists a unique NE at which each user achieves its predefined SINR in multi-cell networks as well. By way of simulation, we evaluate the performance of our algorithm for different pricing and QoS levels, and show how the entry of a new user affects the convergence of the algorithm.

This paper is organized as follows. The problem of joint data rate and power allocation is formulated in Section II, and the pricing mechanism and our proposed algorithm are introduced in Section III, followed by its extension to a multi-cell scenario in Section IV. Practical considerations are addressed in Section V, simulation results are presented in Section VI, and conclusions are in Section VII.

II. Non-cooperative Joint Rate and Power Control Game

In this section, we present the non-cooperative joint data rate and power control game (NJRPCG) in which pricing is not applied. Consider a single cell CDMA network with M active users that are randomly spread in the coverage area. Channel gain from user i to its base station is g_i, whose bandwidth is W. A given user i transmits at data rate r_i with power p_i. SINR of user i at its base station is [4], [16]

$$\gamma_i = \frac{W}{r_i} \frac{g_i p_i}{\sum\limits_{j \neq i} g_j p_j + N_0}, \tag{1}$$

where N_0 denotes noise power, and $\frac{W}{r_i}$ is the processing gain. We rewrite (1) as

$$\gamma_i = \frac{W}{r_i} \frac{p_i}{R_i^{\text{eff}}}, \tag{2}$$

where R_i^{eff} is the effective interference at the receiver of user i, defined by

$$R_i^{\text{eff}} = \frac{\sum\limits_{j \neq i} g_j p_j + N_0}{g_i}. \tag{3}$$

From (1) and (3), it is easy to see that the channel condition for user i depends on the direct channel gain g_i, and on the interference experienced by that user, i.e., $\sum\limits_{j \neq i} g_j p_j + N_0$. Note that in some cases, although direct channel gain of a user may be high, the channel conditions may not be good because of high interference.

For a given modulation type, SINR of a user corresponds to its received QoS level in terms of its bit error rate (BER). As such, for a user to achieve its required QoS, it is necessary to maintain its SINR above a predefined value, i.e., $\gamma_i \geq \hat{\gamma}$. We assume that the transmit power levels and data rates of users are bounded, i.e., $P_i^{\text{min}} \leq p_i \leq P_i^{\text{max}}$, $R_i^{\text{min}} \leq r_i \leq R_i^{\text{max}}$.

The NJRPCG game $\mathcal{G} = \langle \mathcal{M}, \{(\mathcal{P}_i, \mathcal{R}_i)\}, \{u_i\} \rangle$ consists of a set of mobile users $\mathcal{M} = \{1, \cdots, M\}$ that act as players. The strategy space is $\{(\mathcal{P}_i, \mathcal{R}_i)\}$, where $\mathcal{P}_i$ is the power strategy set and $\mathcal{R}_i$ is the data rate strategy set, and u_i is the utility function for user i. The utility of each user is a function of both transmit power levels and data rates of all users, i.e., $u_i(\mathbf{p}, \mathbf{r})$. Since the strategy space of each user is two dimensional, analysis of the game is more complicated than those of power control games studied in [3]–[5]. In a NJRPCG, each user maximizes its own utility in a distributed manner by choosing its transmit power level and data rate. The objective of the NJRPCG game $\mathcal{G}$ is

$$\max_{(\mathcal{P}_i, \mathcal{R}_i)} \; u_i(p_i, \mathbf{p}_{-i}, r_i, \mathbf{r}_{-i}), \qquad \forall i \in \mathcal{M}, \tag{4}$$

where $\mathbf{p}_{-i}$ and $\mathbf{r}_{-i}$ are transmit power and data rate strategies of all users other than user i, respectively. The outcome of the game is the NE at which no user can increase its utility by unilaterally changing its strategy, given that other users' strategies are fixed. Formally, NE is defined as follows.

Definition 1: Transmit power level vector $\mathbf{p}^*$ and data rate vector $\mathbf{r}^*$ is NE of the NJRPCG game $\mathcal{G}$ if

$$u_i(p_i^*, \mathbf{p}_{-i}^*, r_i^*, \mathbf{r}_{-i}^*) \geq u_i(p_i, \mathbf{p}_{-i}^*, r_i, \mathbf{r}_{-i}^*), \qquad \forall p_i \in \mathcal{P}_i, \qquad \forall r_i \in \mathcal{R}_i, \qquad \forall i \in \mathcal{M}, \tag{5}$$

Another definition of NE is based on the best response notion, which is a set valued function $\text{br}_i : (\mathcal{P}_{-i}, \mathcal{R}_{-i}) \rightarrow (\mathcal{P}_i, \mathcal{R}_i)$ defined by

$$\text{br}_i(\mathbf{p}_{-i}, \mathbf{r}_{-i}) = \{p_i \in \mathcal{P}_i, r_i \in \mathcal{R}_i : u_i(p_i, \mathbf{p}_{-i}, r_i, \mathbf{r}_{-i}) \geq u_i(\hat{p}_i, \mathbf{p}_{-i}, \hat{r}_i, \mathbf{r}_{-i}), \forall \hat{p}_i \in \mathcal{P}_i, \hat{r}_i \in \mathcal{R}_i\}. \tag{6}$$

Note that, $\mathcal{P}_{-i} = \prod_{j \neq i} \mathcal{P}_j$ and $\mathcal{R}_{-i} = \prod_{j \neq i} \mathcal{R}_j$. As such, vectors $\mathbf{p}^*$ and $\mathbf{r}^*$ constitute NE of the game if and only if for all users $(p_i^*, r_i^*) \in \text{br}_i(\mathbf{p}_{-i}^*, \mathbf{r}_{-i}^*)$.

In power control, a variety of utility functions can be used. In [7], a utility that corresponds to energy efficiency is used, i.e., utility is defined as the ratio of the transmitted information (in bits) per unit of consumed power. In [5], a sigmoid like function of SINR is used for utility. However, for the problem at hand, as stated earlier, due to the interaction between data rate and transmit power, such utilities would lead to multiple Nash equilibria.

We choose the utility function in such a way that the following three requirements would be satisfied:

1. Each user aims to achieve higher SINRs.
2. Each user aims to attain higher data rates.
3. When the channel is bad and/or when interference is high, each user should increase its transmit power level and/or decrease its data rate.

Note that the above three requirements may be in conflict with each other. For example, from (1), one can easily see that the above Requirements 1 and 2 are in conflict. In addition, although each user wants to transmit at higher data rates, adapting to its channel conditions may decrease its data rate, meaning that Requirements of 2 and 3 may not be simultaneously adhered to. As such, we may have to trade-off between the above requirements. On the other hand, for joint data rate and power control, the utility is a function of the user's transmit power and its data rate, and may not be directly a function of SINR. Besides, in a game theoretic framework, each user aims to maximize its utility function, and the game will settle at its NE, if one exits. Hence, each user will achieve an SINR computed by transmit power levels and data rates of users at the NE. In addition, the achieved SINR of each user depends on the definition of utility.

We begin by considering the utility of each use as a logarithmic function of its SINR, as this functions is commonly used for power control [3] and for joint data rate and power control [17] schemes, i.e.,

$$u_i^1 = \log(1 + k\gamma_i), \tag{7}$$

where k is an adjustable parameter. Since the utility in (7) is an increasing function of SINR, maximizing this utility in terms of data rate and power level will result in the user's SINR γ_i. This is in line with Requirement 1 above. To maximize the utility in (7), each user transmits at a high power level and a low data rate, which contradicts the second point above. To alleviate this, we add a new term to (7), which is a logarithmic function of data rate r_i, i.e.,

$$u_i^2 = \log(k' r_i), \tag{8}$$

where k' is an adjustable parameter. From (7) and (8), the utility of each user is

$$u_i(p_i, \mathbf{p}_{-i}, r_i, \mathbf{r}_{-i}) = \log(1 + k\frac{W}{r_i}\frac{g_i p_i}{\sum\limits_{j\neq i} g_j p_j + N_0}) + \log(k' r_i). \tag{9}$$

Note that the utility of user i does not depend on data rates of other users, i.e., on $\mathbf{r}_{-i}$, but to follow the general rule, we write it in this manner. We rewrite (9) as

$$u_i(p_i, \mathbf{p}_{-i}, r_i, \mathbf{r}_{-i}) = \log(k_1 r_i + k_2 \frac{p_i}{R_i^{\text{eff}}}). \tag{10}$$

where $k_1 = k'$ and $k_2 = k'kW$. In what follows, we analyze the NJRPCG game (4) for the utility function (10).

Theorem 1: There exists a unique NE in the NJRPCG game (4).

Proof: The utility function of user i, i.e., (10), is an increasing function of (p_i, r_i), meaning that higher values of transmit power p_i and data rate r_i lead to a higher utility value. This means that in order to maximize the utility, each user transmits at its maximum transmit power and maximum data rate. As such, there exists a unique NE at which each user transmits at its maximum transmit power and maximum data rate. ■

At NE of (4), each user transmits at maximum data rate and maximum power, regardless of channel conditions. However, in general, the data rate should be so chosen to satisfy Shannon capacity. For the game involving (4), the chosen data rate that maximizes the utility may be above the Shannon capacity. In such cases, satisfying Shannon capacity entails complications that involve both the user and the base station. To avoid such complications, we use pricing for the games in Sections III and IV to improve a user's SINR, which in turn would result in a higher Shannon capacity, meaning that the chosen data rates for those users that with pricing can achieve their target SINRs are feasible, i.e., below Shannon capacity. If the strategy space is bounded, some users may not achieve their target SINRs when pricing is used, and should be removed as their data rates would be above Shannon capacity. Morover, as stated earlier, each user may adapt its transmit power and data rate to its channel conditions (Requirement 3 in Section II). In the next section, we present a pricing mechanism and show that if the strategy space is unbounded, at NE of the game with pricing, each user will achieve its predefined SINR.

III. Non-Cooperative Joint Rate and Power Control Game with Pricing

Transmitting at maximum data rate and maximum power is not always useful, since the transmit power of each user is considered as interference to other users. To control the selfish behavior of each user and adapt its transmit power and/or data rate to the channel condition, we adjust the parameters of utility

function and use pricing. In doing so, we present a joint data rate and transmit power control game with pricing (NJRPCGP), show that the corresponding distributed algorithm converges to the unique NE of the game, and demonstrate that each user achieves its predefined SINR. Since the strategy space of users are transmit power levels and data rates, our pricing depends on both of these values for each user. The multi-dimensionality of the strategy space of users means that we have many choices for the pricing function, such as the weighted sum of users' power levels and data rates, and their multiplications.

Here, for each users, we use the weighted sum of its squared data rate and its squared transmit power as the pricing function. Our choice of the pricing function has the following merits: First, the utility function remains concave, which is essential for the game's analysis; and second, the squared function is more sensitive to variations in its parameters than a linear function. This means that a user's choice of its strategies highly affects the value of its pricing factor. We will show in our simulations that users who transmit at higher power levels and higher data rates will reduce their transmit power level and data rate more than those users who transmit at lower power levels and at lower data rates when pricing is increased. The utility of each user is

$$u_i = \log(\alpha_2 r_i + \alpha_1 p_i) - \frac{\lambda}{2}(\frac{\alpha_2}{\alpha_1} r_i^2 + \frac{\alpha_1}{\alpha_2} p_i^2), \tag{11}$$

where α_1 and α_2 are adjustable parameters and λ is the pricing factor. The utility (11) has the property that for a given value of the utility function and a fixed λ, when $\frac{\alpha_2}{\alpha_1}$ increases, we must decrease the data rate and/or increase the power level to keep the utility fixed. This can be used to incorporate Requirement 3 in Section II into the utility function (11). To this end, we introduce the effective interference R_i^{eff} into the utility by

$$u_i = \log(\alpha_2 R_i^{\text{eff}} r_i + \alpha_1 p_i) - \frac{\lambda}{2}(\frac{\alpha_2}{\alpha_1} R_i^{\text{eff}} r_i^2 + \frac{\alpha_1}{\alpha_2} \frac{1}{R_i^{\text{eff}}} p_i^2). \tag{12}$$

In this way, when the effective interference R_i^{eff} for user i increases, that user decreases its data rate and/or increases its power. We will show that the values of α_1 and α_2 are important in the actual SINR for each user. For the above utility, the objective of the NJRPCGP game is

$$\max_{(P_i, R_i)} \; u_i(p_i, \mathbf{p}_{-i}, r_i, \mathbf{r}_{-i}), \forall i \in \mathcal{M}. \tag{13}$$

Theorem 2: There exists a NE in the above NJRPCGP game.

Proof: One can easily show that utility u_i is a jointly concave function of (p_i, r_i) by forming its second derivatives, i.e.,

$$\frac{\partial^2 u_i}{\partial p_i^2} = -\frac{\alpha_1^2}{(\alpha_1 p_i + \alpha_2 R_i^{\text{eff}} r_i)^2} - \lambda \frac{\alpha_1}{\alpha_2} \frac{1}{R_i^{\text{eff}}}, \tag{14}$$

$$\frac{\partial^2 u_i}{\partial r_i^2} = -\frac{(\alpha_2 R_i^{\text{eff}})^2}{(\alpha_1 p_i + \alpha_2 R_i^{\text{eff}} r_i)^2} - \lambda \frac{\alpha_2}{\alpha_2} R_i^{\text{eff}}, \tag{15}$$

$$\frac{\partial^2 u_i}{\partial p_i \partial r_i} = -\frac{\alpha_1 \alpha_2 R_i^{\text{eff}}}{(\alpha_1 p_i + \alpha_2 R_i^{\text{eff}} r_i)^2}. \tag{16}$$

It is obvious that inequalities in $\frac{\partial^2 u_i}{\partial p_i^2} \leq 0$, $\frac{\partial^2 u_i}{\partial r_i^2} \leq 0$, and $\frac{\partial^2 u_i}{\partial p_i^2}\frac{\partial^2 u_i}{\partial r_i^2} - (\frac{\partial^2 u_i}{\partial p_i \partial r_i})^2 \geq 0$ are strict. Therefore, the utility function is strictly concave on (p_i, r_i). Besides, the utility function is continuous in $(\mathbf{p}, \mathbf{r})$. Since the strategy space $(\mathcal{P}_i, \mathcal{R}_i)$ is a compact, convex, and nonempty subset of $\mathbb{P}^M \times \mathbb{R}^M$, where $\mathbb{P}^M$ and $\mathbb{R}^M$ are M-dimensional Euclidean space of real numbers, the proof follows. ■

To obtain the best response function for each user i, we use the first derivative of u_i with respect to (p_i, r_i), and write

$$\begin{cases} \frac{\partial u_i}{\partial p_i} = 0 \ \longrightarrow \frac{\alpha_2 R_i^{\text{eff}}}{\alpha_1 p_i + \alpha_2 R_i^{\text{eff}} r_i} - \lambda p_i = 0, \\ \frac{\partial u_i}{\partial r_i} = 0 \ \longrightarrow \frac{\alpha_1}{\alpha_1 p_i + \alpha_2 R_i^{\text{eff}} r_i} - \lambda r_i = 0. \end{cases} \tag{17}$$

Rewriting (17), we obtain the following system of equations

$$\begin{cases} \alpha_1 \lambda p_i^2 + \alpha_2 \lambda R_i^{\text{eff}} p_i r_i = \alpha_2 R_i^{\text{eff}}, \\ \alpha_1 \lambda p_i r_i + \alpha_2 \lambda R_i^{\text{eff}} r_i^2 = \alpha_1. \end{cases} \tag{18}$$

By solving (18), we obtain the following positive values for (p_i, r_i)

$$\begin{cases} p_i = \sqrt{\frac{1}{2}\frac{\alpha_2}{\alpha_1}\frac{R_i^{\text{eff}}}{\lambda}}, \\ r_i = \sqrt{\frac{1}{2}\frac{\alpha_1}{\alpha_2}\frac{1}{\lambda R_i^{\text{eff}}}}. \end{cases} \tag{19}$$

Remark 1: From (19), one can observe that when the channel of user i is bad, i.e., when its channel gain is low and/or when interference from other users is high, that user increases its transmit power and/or decreases its data rate.

Using (19), we propose the following iterative algorithm for updating (p_i, r_i)

$$(\mathbf{p}^{n+1}, \mathbf{r}^{n+1}) = (\mathbf{I}^{\text{p}}(\mathbf{p}^n, \mathbf{r}^n), \mathbf{I}^{\text{r}}(\mathbf{p}^n, \mathbf{r}^n)), \tag{20}$$

where

$$\begin{cases} I_i^{\text{p}}(\mathbf{p}, \mathbf{r}) = \sqrt{\frac{1}{2}\frac{\alpha_2}{\alpha_1}\frac{R_i^{\text{eff}}}{\lambda}}, \\ I_i^{\text{r}}(\mathbf{p}, \mathbf{r}) = \sqrt{\frac{1}{2}\frac{\alpha_1}{\alpha_2}\frac{1}{\lambda R_i^{\text{eff}}}}. \end{cases} \tag{21}$$

Note that if (20) converges, it will converge to the fixed points $(\mathbf{I}^{\text{p}}(\mathbf{p}, \mathbf{r}), \mathbf{I}^{\text{r}}(\mathbf{p}, \mathbf{r}))$. Therefore, for the algorithm to converge, it is necessary to have a fixed point. In general, the fixed point (and the NE of the game) may not be unique. In such cases, one can devise an algorithm and derive its initial conditions such that the algorithm converges to a specific NE as was done in the S-modular game in [4] and the joint rate and power control game in [14]. Otherwise, the algorithm may toggle between the NEs of the

game and would not converge. However, if the fixed point (and the NE) is unique, the algorithm would indeed converge to its unique fixed point. Now, the remaining questions are under what conditions, the fixed points exist, and if the fixed point is unique.

Remark 2: Note that both I_i^{p} and I_i^{r} depend only on λ, α_1, α_2, and R_i^{eff}, all of which are either locally available or can be broadcasted by the base station to all users. In other words, each user i updates its data rate and its transmit power according to (21) in a distributed manner. In addition, this means that the feedback overhead of the proposed scheme is low and are comparable to the information needed by the well known power control algorithms such as TPC [2]. Moreover, since each user updates its transmit power and its data rate according to (21), our scheme needs minimal calculations.

Remark 3: Considering (21), one can observe that the data rate and the transmit power of a given user depend on R_i^{eff}, which can be computed from (3). In other words, the data rate of user i does not have any impact on transmit power levels or on data rates of other users. Therefore, we can omit the variable $\mathbf{r}$ from the arguments of $\mathbf{I}^{\mathrm{p}}$ and $\mathbf{I}^{\mathrm{r}}$. Besides, for convergence, it is sufficient to prove the existence and uniqueness of the fixed point in the transmit power update, i.e., $\mathbf{p}^{n+1} = \mathbf{I}^{\mathrm{p}}(\mathbf{p}^n)$.

From *Remark 3*, we rewrite the transmit power update function $\mathbf{I}^{\mathrm{p}}(\mathbf{p})$ as

$$I_i^{\mathrm{p}}(\mathbf{p}) = \sqrt{\frac{1}{2}\frac{\alpha_2}{\alpha_1}\frac{1}{\lambda}(\sum_{j\neq i}\frac{g_j}{g_i}p_j + \frac{N_0}{g_i})}. \tag{22}$$

Note that when I_i^{p} has a unique fixed point, the iterative transmit power update will globally converge to its fixed point. To prove the existence of a fixed point, we use the following theorem.

Theorem 3 (Brouwer's Fixed Point Theorem) [18]: Let $\hat{\mathcal{P}} \subseteq \mathbb{R}^M$ be compact and convex, and $F : \hat{\mathcal{P}} \longrightarrow \hat{\mathcal{P}}$ be a continuous function. There exists a $\mathbf{p} \in \hat{\mathcal{P}}$ such that $\mathbf{p} = F(\mathbf{p})$.

Using Theorem 3, we prove the existence of a fixed point for $\mathbf{I}^{\mathrm{p}}(\mathbf{p})$ in the following theorem.

Theorem 4: The function $\mathbf{I}^{\mathrm{p}}(\mathbf{p})$ has a fixed point, i.e., there exists a transmit power vector $\mathbf{p}^*$ such that $\mathbf{p}^* = \mathbf{I}^{\mathrm{p}}(\mathbf{p}^*)$.

Proof: Since the power update function $\mathbf{I}^{\mathrm{p}}(\mathbf{p})$ is continuous, we need to show that there exists a set $\hat{\mathcal{P}} \subset \mathbb{R}^M$ such that for each $\mathbf{p} \in \hat{\mathcal{P}}$, we have $\mathbf{I}^{\mathrm{p}}(\mathbf{p}) \in \hat{\mathcal{P}}$. From (22), one can observe that for all $\mathbf{p} \geq \mathbf{0}$ we have $I_i^{\mathrm{p}}(\mathbf{p}) \geq l_i = \sqrt{\frac{1}{2}\frac{\alpha_2}{\alpha_1}\frac{1}{\lambda}(\frac{N_0}{g_i})}$. We define $\underline{l} = \min_i l_i$, $c_j = \max_i \frac{1}{2}\frac{\alpha_2}{\alpha_1}\frac{1}{\lambda}\frac{g_j}{g_i}$, and $\overline{c} = \max(\max_i c_i, \max_i l_i)$. As such, $x^2 = (M-1)\overline{c}x + \overline{c}$ is well defined. This equation always has two roots, and the larger one is denoted by $\overline{x}$. We define the set $\hat{\mathcal{P}} = \{\mathbf{p} : \underline{l} \leq p_i \leq \overline{z}\}$ where $\overline{z} > \overline{x}$. Note that for each $\mathbf{p} \in \hat{\mathcal{P}}$, we have $\mathbf{I}^{\mathrm{p}}(\mathbf{p}) \in \hat{\mathcal{P}}$. Therefore, we have a continuous function $\mathbf{I}^{\mathrm{p}}(\mathbf{p})$ such that $\mathbf{I}^{\mathrm{p}} : \hat{\mathcal{P}} \longrightarrow \hat{\mathcal{P}}$. From Theorem 3, the function $\mathbf{I}^p$ has a fixed point. ■

Remark 4: From Theorem 4, it follows that the power update function $\mathbf{I}^{\mathrm{p}}(\mathbf{p})$ always has a fixed point, and if it is unique, it globally converges to the unique fixed point. This is in contrast to TPC [2] that only under some conditions has a fixed point [19]. Moreover, this means that we do not need the compactness assumption as in Theorem 2, i.e., the strategy space of users can be unbounded. This property does not hold in [12]–[14].

In addition to the existence of a fixed point, in Theorem 5, we prove that the fixed point is unique, which guarantees that the algorithm globally converges to this unique NE.

Theorem 5: The fixed point of $\mathbf{I}^{\mathrm{p}}(\mathbf{p})$ is unique.

Proof: We use the notion of standard functions defined in [20]. A standard function $\mathbf{I}(\mathbf{p})$ has the following three properties for all $\mathbf{p} \geq \mathbf{0}$:

1. Positivity: $\mathbf{I}(\mathbf{p}) > \mathbf{0}$
2. Monotonicity: if $\mathbf{p} > \mathbf{p}'$, then $\mathbf{I}(\mathbf{p}) \geq \mathbf{I}(\mathbf{p}')$
3. Scalability: for all $a > 1$, $a\mathbf{I}(\mathbf{p}) \geq \mathbf{I}(a\mathbf{p})$

In Theorem 1 in [20], it is shown that when a standard function has a fixed point, that fixed point is unique. One can easily show that $\mathbf{I}^{\mathrm{p}}(\mathbf{p})$ is a standard function. From Theorem 4, we know that $\mathbf{I}^{\mathrm{p}}(\mathbf{p})$ has a fixed point, and thus, the fixed point is unique. ■

From Theorems 4 and 5 together with Remark 3, it follows that the iterative updating function in (20) converges to a unique fixed point. Therefore, the output of the joint data rate and transmit power control game with pricing is a unique NE, and the distributed data rate and transmit power updates converge to this unique NE. In what follows, we prove an interesting property of NE of NJRPCGP game in (13).

Theorem 6: At NE, data rate and transmit power of each user are related via

$$\frac{1}{r_i}\frac{p_i}{R_i^{\mathrm{eff}}} = \frac{\alpha_2}{\alpha_1}. \tag{23}$$

Proof: The proof follows from (19). ■

Theorem 6 has the following interesting notion. We rewrite (2) as

$$\gamma_i = \frac{W}{r_i}\frac{p_i}{R_i^{\mathrm{eff}}} = \frac{\alpha_2}{\alpha_1}W. \tag{24}$$

Therefore, when the required SINR for user i is γ_i in (24), that user will attain its SINR at NE. We can set different values of α_1 and α_2 for different users, resulting in different values of SINR. Note that the SINR value $\frac{\alpha_2}{\alpha_1}W$ is the SINR that any user can achieve by maximizing its utility. In Theorem 6, this SINR is predefined as $\frac{\alpha_2}{\alpha_1}W$, which can be set by the base station and/or the service provider based on the service and the user's quality of service.

In the above, we assumed that the convergence point of the iterative updating function (21) is in the strategy space of each user, and showed that under this assumption, the distributed joint data rate and transmit power allocation algorithm always converges to a point at which each user achieves its target SINR. But, this assumption may not be valid in general. In reality, bounds on users' data rates and power levels as well as channel conditions can move the convergence point of the iterative updating function (21) outside the strategy space defined for each user. In what follows, we analyze this case.

Consider the utility and the game formulation in (12) and (13), respectively, and the strategy space of each user as in Section II. When the convergence point of the iterative updating function (21), i.e., (p_i, r_i) is such that $P_i^{\text{min}} \leq p_i \leq P_i^{\text{max}}$, $R_i^{\text{min}} \leq r_i \leq R_i^{\text{max}}$, the solution of the game is (19) and the iterative updating is (20). When data rate of a user reaches its upper or lower bounds, the solution for data rate is that bound (i.e., $r_i = R_i^{\text{bound}}$ where $R_i^{\text{bound}} = R_i^{\text{min}}$ if it reaches the lower bound, and $R_i^{\text{bound}} = R_i^{\text{max}}$ if it reaches the upper bound). To obtain the power level, in (12) we replace r_i with R_i^{bound}, take the derivative with respect to p_i, and write

$$\alpha_1 \lambda p_i^2 + \alpha_2 \lambda R_i^{\text{eff}} R_i^{\text{bound}} p_i - \alpha_2 R_i^{\text{eff}} = 0. \tag{25}$$

The iterative transmit power update function $\mathbf{I}^{\text{p}}$ is obtained from the positive root of (25) as

$$I_i^{\text{p}}(\mathbf{p}) = \frac{-\alpha_2 \lambda R_i^{\text{eff}} R_i^{\text{bound}} + \sqrt{(\alpha_2 \lambda R_i^{\text{eff}} R_i^{\text{bound}})^2 + 4\alpha_1 \alpha_2 \lambda R_i^{\text{eff}}}}{2\alpha_1 \lambda}. \tag{26}$$

The power level of a user is bounded by its upper or lower limits. When transmit power of a user reaches its upper or lower bounds, to obtain the data rate, we replace p_i with P_i^{bound} in (12), take its derivative with respect to r_i, and write

$$\alpha_2 \lambda R_i^{\text{eff}} r_i^2 + \alpha_1 \lambda P_i^{\text{bound}} r_i - \alpha_1 = 0. \tag{27}$$

The iterative data rate update function $\mathbf{I}^{\text{r}}$, is obtained from the positive root of (27) as

$$I_i^{\text{r}}(\mathbf{p}) = \frac{-\alpha_1 \lambda P_i^{\text{bound}} + \sqrt{(\alpha_1 \lambda P_i^{\text{bound}})^2 + 4\alpha_1 \alpha_2 \lambda R_i^{\text{eff}}}}{2\alpha_2 \lambda R_i^{\text{eff}}}. \tag{28}$$

Since (26) is a standard function, one can easily prove the following theorem.

Theorem 7: Consider the NJRPCGP game (13). The distributed joint data rate and transmit power control algorithm will always converge to the unique NE of the game.

When the convergence point of the iterative updating function (21), i.e., (p_i, r_i) is within the strategy space of users, all users will achieve their target SINRs. This is not true when this point is outside the strategy space of users, resulting in some users reaching their lower bound and/or upper bound on data rate and/or transmit power. Note that a user achieves a higher SINR by increasing its transmit power

and/or by decreasing its data rate. Therefore, the achieved SINR of those users who reach their lower bound on data rate and/or upper bound on transmit power is below their target SINR. Such users consume network resources without achieving their required QoS, and cause interference to other users as well. To deal with such cases, depending on service provisioning policies by the network provider, either these users should be removed from the network [21], [22], or pricing should be changed [23]. Interference caused by those users that cannot attain their target SINRs can be reduced by removing them from the network one by one until all remaining users achieve their target SINR. Interference caused by those users whose achieved SINRs is higher than their target SINRs can be reduced by forcing such users to reduce their transmit power levels and/or data rates.

The non-cooperative joint data rate and power control game with pricing is stated below.

NJRPCGP Algorithm:

1. each user begins transmitting at power level $p_i(t_0)$ and data rate $r_i(t_0)$ constituting the power vector $\mathbf{p}(t_0)$ and the data rate vector $\mathbf{r}(t_0)$.
2. At each iteration time t_k, each user updates its the transmit power level and its data rate according to (21):
   a. When $P_i^{\text{min}} \leq p_i \leq P_i^{\text{max}}$ and $R_i^{\text{min}} \leq r_i \leq R_i^{\text{max}}$, we set $p_i(t_k) = p_i$ and $r_i(t_k) = r_i$.
   b. When $P_i^{\text{min}} \leq p_i \leq P_i^{\text{max}}$, and $r_i \leq R_i^{\text{min}}$ or $R_i^{\text{max}} \leq r_i$, we update $p_i(t_k)$ from (26) and set $r_i(t_k) = R_i^{\text{min}}$ or $r_i(t_k) = R_i^{\text{max}}$, respectively.
   c. When $p_i \leq P_i^{\text{min}}$ or $P_i^{\text{max}} \leq p_i$ and $R_i^{\text{min}} \leq r_i \leq R_i^{\text{max}}$, we set $p_i(t_k) = P_i^{\text{min}}$ or $p_i(t_k) = P_i^{\text{max}}$, respectively, and update $r_i(t_k)$ from (28).
3. When $\max_i \left(\|p_i(t_k) - p_i(t_{k-1})\| + \|r_i(t_k) - r_i(t_{k-1})\| \right) \leq \delta$, we stop; else we go to Step 2 above.

IV. Base Station Assignment

In this section, we extend our proposed scheme to a multi-cell network in which active users communicate through B base stations. The base station assigned to user i is a_i, and $g_{a,i}$ denotes the channel gain from the transmitter of user i to base station a. Each user transmits at data rate r_i at power p_i. The SINR of user i at its base station a_i is

$$\gamma_{a_i,i} = \frac{W}{r_i} \frac{g_{a_i,i} p_i}{\sum\limits_{j \neq i} g_{a_i,j} p_j + N_0}, \tag{29}$$

where we assume noise power N_0 is the same for all base stations.

In a multi-cell network, base station assignment must also be considered. We incorporate base station assignment into our joint data rate and transmit power level scheme proposed in Section III, i.e., each user chooses its base station in a distributed manner in addition to setting its transmit power level and data rate. Since utility of each user is a decreasing function of its effective interference R_i^{eff}, we use the value of R_i^{eff} as a measure for base station assignment. The objective of the non-cooperative joint data rate, transmit power level, and base station assignment game with pricing (NJRPCGPB) is

$$\max_{(P_i,R_i)} \; u_{a_i,i}(p_i,\mathbf{p}_{-i},r_i,\mathbf{r}_{-i}), \forall i \in \mathcal{M}, \tag{30}$$

where

$$a_i = \text{argmin}_a \; R_{a,i}^{\text{eff}}, \tag{31}$$

and

$$u_{a,i} = \log(\alpha_2 R_{a,i}^{\text{eff}} r_i + \alpha_1 p_i) - \frac{\lambda}{2}(\frac{\alpha_2}{\alpha_1} R_{a,i}^{\text{eff}} r_i^2 + \frac{\alpha_1}{\alpha_2}\frac{1}{R_{a,i}^{\text{eff}}} p_i^2), \tag{32}$$

where $R_{a,i}^{\text{eff}}$ is defined in (3). Dynamic base station assignment according to (31) means that $R_{a_i,i}^{\text{eff}} \leq R_{a,i}^{\text{eff}}$ is satisfied for all base station assignments. Inverting this inequality and multiplying both sides by $\frac{p_i}{r_i}$, we get

$$\frac{p_i}{r_i}\frac{1}{R_{a_i,i}^{\text{eff}}} \geq \frac{p_i}{r_i}\frac{1}{R_{a,i}^{\text{eff}}}. \tag{33}$$

Equation (33) has the following interpretation. For a fixed value of $\frac{p_i}{r_i}$, each user is assigned to a base station in which its SINR is the highest compared to all other choices for base station. When target SINR is predefined via the values of α_1, α_2 and λ before the game starts, each user is assigned to a base station for which it transmits at a lower power level and a higher data rate as compared to all other choices for base station.

Note that for each value of $R_{a,i}^{\text{eff}}$, the utility function of each user is a jointly concave function of (p_i, r_i), and hence $u_{a_i,i}$ is a concave function, and the game (30) is well defined. From (31), (32), and (21), one can derive the power and data rate update functions when base station is assigned. We propose the following iterative algorithm for updating (p_i, r_i):

$$(\mathbf{p}^{n+1}, \mathbf{r}^{n+1}) = (\mathbf{I}^{\text{p}}(\mathbf{p}^n, \mathbf{r}^n), \mathbf{I}^{\text{r}}(\mathbf{p}^n, \mathbf{r}^n)), \tag{34}$$

where

$$\begin{cases} I_i^{\text{p}}(\mathbf{p},\mathbf{r}) = \min_a I_{a,i}^{\text{p}}(\mathbf{p},\mathbf{r}), \\ I_i^{\text{r}}(\mathbf{p},\mathbf{r}) = \max_a I_{a,i}^{\text{r}}(\mathbf{p},\mathbf{r}), \end{cases} \tag{35}$$

and

$$\begin{cases} I^{\mathrm{p}}_{a,i}(\mathbf{p},\mathbf{r}) = \sqrt{\frac{1}{2}\frac{\alpha_2}{\alpha_1}\frac{R^{\mathrm{eff}}_{a,i}}{\lambda}}, \\ I^{\mathrm{r}}_{a,i}(\mathbf{p},\mathbf{r}) = \sqrt{\frac{1}{2}\frac{\alpha_1}{\alpha_2}\frac{1}{\lambda R^{\mathrm{eff}}_{a,i}}}. \end{cases} \tag{36}$$

From Theorem 5 in [20], the power update function in (35) is a standard function. Hence all results in Section VI for the game (13) also hold for the game (30). In particular, we have the following theorem.

Theorem 8: The game (30) has a unique NE to which our distributed algorithm (34) converges. When the convergence point of the iterative updating function is within the strategy space of users, each user will achieve its target SINR.

The non-cooperative joint rate, power, and base station assignment game with pricing is stated below.

NJRPCGPB algorithm:

1. each user connects randomly to one base station and begins transmitting at the power level $p_i(t_0)$ and the data rate $r_i(t_0)$ constituting the rate vector $\mathbf{r}(t_0)$ and power vector $\mathbf{p}(t_0)$.
2. At each iteration time t_k, each user updates its base station to which it connects, i.e., a_i, according to (31).
3. Each user updates the transmit power level and rate according to (36):
   a. When $P_i^{\min} \le p_i \le P_i^{\max}$ and $R_i^{\min} \le r_i \le R_i^{\max}$, we set $p_i(t_k) = p_i$ and $r_i(t_k) = r_i$.
   b. When $P_i^{\min} \le p_i \le P_i^{\max}$, and $r_i \le R_i^{\min}$ or $R_i^{\max} \le r_i$, we update $p_i(t_k)$ from (26) and set $r_i(t_k) = R_i^{\min}$ or $r_i(t_k) = R_i^{\max}$, respectively.
   c. When $p_i \le P_i^{\min}$ or $P_i^{\max} \le p_i$ and $R_i^{\min} \le r_i \le R_i^{\max}$, we set $p_i(t_k) = P_i^{\min}$ or $p_i(t_k) = P_i^{\max}$, respectively, and update $r_i(t_k)$ from (28).
4. When $\max_i \left(\|p_i(t_k) - p_i(t_{k-1})\| + \|r_i(t_k) - r_i(t_{k-1})\|\right) \le \delta$, we stop; else we go to Step 2 above.

Note that the NJRPCGPB algorithm is similar to the NJRPCGP algorithm except in Step 2, where base stations are assigned to users.

V. Practical Considerations

In this section, we explain some practical issues concerning system parameters, message passing, and discrete data rates in the proposed algorithms.

A. System Parameters

There are three parameters in the utility function, namely α_1, α_2, and λ; each with a different impact on data rates and power levels of users. In Theorem 6, the impacts of α_1 and α_2 are shown. As stated

in (24), at the NE of the game (games in Sections III and IV), depending on the values of α_1 and α_2, each user will achieve a specific SINR. This means that by setting the values of α_1 and α_2, the target SINR for each user is set. In general, the values of α_1 and α_2 can be set by the base stations and/or by the service provider based on the service and the user's required quality of service. As such, depending on channel conditions, each users chooses its transmit power and data rate to achieve its SINR. As an example, suppose that in a network with $W = 10^6$ Hz bandwidth, the required quality of service for user i in terms of its SINR is $\gamma_i = 20$. Considering (24), we have $\frac{\alpha_2}{\alpha_1} = \frac{\gamma_i}{W} = 2 \times 10^{-5}$. In this case, all values of α_1 and α_2 that satisfy $\frac{\alpha_2}{\alpha_1} = 2 \times 10^{-5}$, e.g., $\alpha_1 = 10^6$ and $\alpha_2 = 20$, or $\alpha_1 = 15$ and $\alpha_2 = 3 \times 10^{-4}$ are acceptable. This means that only the ratio $\frac{\alpha_2}{\alpha_1}$ is important, rather than the values of α_1 and α_2.

The pricing λ can be different for each user, and affects its transmit power and data rate. Increasing λ would decrease the transmit power and the data rate, and decreasing λ would result in the opposite. Depending on channel conditions and strategy bounds, the convergence point of the algorithm may not be within the strategy space, meaning that at NE, some users may not achieve their predefined SINR. By increasing λ, some users decrease their transmit power, and it may possible that more users achieve their target SINR. However, finding the optimal value of λ is not easy, and in many cases a heuristic approach is needed to tune the pricing.

Depending on problem formulation and objectives, one can include many factors into pricing. As the number of users increases, interference also increases. This is undesirable, as it may cause some users not to achieve their SINRs. However, one can include the number of users in the pricing, i.e., $\lambda_i = cM$, so that when the number of users increases, pricing would increase as well. The direct path gain of a user to its receiver is also an option. By proper inclusion of this parameter into pricing, one can achieve better results. For example, if pricing is an increasing function of the direct channel gain $\lambda_i = cg_i$, the algorithm favors those users whose channel gains are low. In this way, one can improve fairness. On the other hand, if pricing is a decreasing function of the direct channel gain $\lambda_i = \frac{c}{g_i}$, the algorithm favors those users whose channel gains are high. In our case that we wish to achieve predefined SINRs at NE, we include the target SINR into pricing similar to the case of direct channel gains. This would make the algorithm favorable to users with low SINR or high SINR, i.e., $\lambda_i = c\frac{\alpha_2}{\alpha_1}$ or $\lambda_i = c\frac{\alpha_1}{\alpha_2}$, respectively.

For the analysis in Section IV, we cannot choose all of the above mentioned values for λ_i, as the iterative power update function in (35) may not be a standard function. To guarantee that the power update function is a standard function, the value of λ_i must be the same for all base stations. This means that the choice of $\lambda_i = cg_{a,i}$ or $\lambda_i = \frac{c}{g_{a,i}}$ may prevent the power update function to be a standard one, and could not be used, since the value of $g_{a,i}$ may be different for each base station a.

Choosing the value of c is not easy, and one may use heuristics to choose its value. In our case, the achievable SINR at NE is predefined, and based on channel conditions and strategy bounds, the convergence point of the algorithm may be out of the strategy space. In such cases, users are divided into 3 categories: users that achieve SINRs below their target SINRs, users that achieve their target SINRs, and users that achieve SINRs above their target SINRs. In our case, only users in the first category do not achieve their required QoS. As pricing increases, the strategy chosen by users tends towards the lower bound in the strategy space; and when pricing decreases, users' strategy tends towards the upper bound in the strategy space. An increase in pricing makes it more likely that all users achieve their target SINRs. Hence, when there are some category 1 users, pricing should be increased by the value of Δc. This should be repeated until all users achieve their target SINRs. In this way, the least value of pricing at which all users achieve their target SINR is selected, meaning that users are allowed to utilize more resources so long as all users can achieve their target SINRs. Note that further increases in pricing will allow entry of new users, as network resources are not completely utilized. However, when increased pricing is not effective, some users may have to be removed.

B. Message Passing

In our proposed algorithm, for each user to update its transmit power and data rate, it needs to know the value of R_i^{eff}, which can be obtained from the value of direct channel gain g_i and the total received power from all users plus noise power at the base station, i.e., $\sum\limits_j g_j p_j + N_0$. It is assumed that the base station broadcasts the value of $\sum\limits_j g_j p_j + N_0$ with fixed power. For the value of g_i, either the base station can transmit it to users or the user can estimate it. In the first case, when the number of users is high, the amount of message passing would be high. Estimating the channel gains by users would be simple in the frequency-division duplexed systems, or in time-division duplexed systems. In such cases, users calculate the interference caused to them from the direct channel gain and the total interference plus noise power at the base station, i.e., $\sum\limits_{j\neq i} g_j p_j + N_0$, by subtracting $g_i p_i$ from $\sum\limits_j g_j p_j + N_0$. Note that with the same amount of message passing as for the TPC, we allocate the transmit power as well as the data rate. The amount of information is also the same as in [14], but as we will show in Section VI, the performance of our algorithm is better than that of [14], as our algorithm consumes much less power than [14].

C. Discrete Data Rates

In our framework and proposed algorithm, we assumed that the data rate for each user is a continuous variable, which may not be true in some actual modulation schemes. In most cases, an algorithm developed

for continuous data rates cannot be applied to discrete data rates, as the continuous assumption is crucial for them. In the game theoretic framework in this paper, each user aims to maximize its utility over a continuous domain of strategy space. In such cases, it may be possible to use the nearest data rate from the discrete data rate values. However, in general, one cannot substitute a continuous variable with a discrete one. This is because of the coupling between the data rate and other variables in the system. In such cases, the algorithm may toggle between two points.

In our proposed scheme, as can be seen from (21) and Remark 3, the data rate of each user has no effect on its transmit power. Instead, only the transmit power levels of users are coupled, and convergence of the algorithm depends only on convergence of the power update function. This would make it possible to consider discrete data rates. To do this, each user updates its transmit power and data rate according to (21), and then chooses the nearest lower data rate from the set of discrete rates available to this user. This would guarantee that the achieved SINR is above the target SINR, as evidenced from (23) and (24).

VI. Simulation Results

We now present simulation results for our distributed joint data rate and power control algorithm. The system under study is the uplink of a CDMA network consisting of two base stations and 5 users located at [110, 130, 210, 390, 410] meters from the first base station and $[410, 390, 310, 130, 110]$ meters from the second base station. The channel gain from user i to base station a is as $g_{a,i} = \frac{\xi}{d_{a,i}^{\eta}}$, where $d_{a,i}$ is the distance between user i and base station a, η is the path loss exponent, and ξ models power variations due to shadowing. We set $\eta = 4$ and $\xi = 0.097$ as in [14]. The bandwidth is $W = 10^6$ Hz.

Consider a network consisting of the first base station and the first three users. We obtain transmit power levels and data rate updates (20) for $\alpha_1 = 10^6$ for all users. We set different values for α_2 for different users, i.e., $\alpha_2 = [20, 25, 30]$ for users 1, 2, and 3, respectively. Pricing factor is set to $\lambda = 10^{-4}$. Fig. 1 shows that every user attains its target SINR.

Next, we consider pricing in the range 0.05 to 1 for each user, and set $\alpha_1 = 10^6$ and $\alpha_2 = 20$ for all users so that their target SINRs are equal to 20. Fig. 2 shows that as pricing increases, each user decreases its data rate and transmit power. In addition, as pricing increases, users that transmit with more power and higher data rates decrease their transmit power and data rate more than those whose pricing is less. This, as stated before, is because of the pricing function which is a squared function of transmit power and data rate. Note that all users achieve their target SINRs regardless of the value of pricing.

We also show the impact of bounding the strategy space (data rates and transmit power levels) of users. Here, pricing is $\lambda = 10^{-5}$, and $\alpha_1 = 10^6$, $\alpha_2 = 20$ for all users. Table I shows that user 2 achieves

its target SINR by adjusting its transmit power and data rate. The achieved SINR by user 1 who enjoys a good channel is above its target SINR; whereas user 3 whose channel is bad does not attain its target SINR. When we increase pricing from $\lambda = 10^{-5}$ to $\lambda = 10^{-4}$, all users achieve their target SINRs by appropriate reductions in their transmit power levels and data rates. Next, We set the pricing back to $\lambda = 10^{-5}$ and remove user 3 from the network. Note that the remaining users increase their data rates and reduce their transmit power levels, and the achieved SINR of user 1 is above its target SINR.

We now show what happens when a new user enters the system. The setup is similar to the one for the first example above, but after a time period, a new user (user 4) enters the system. This user is located 130 meters from the base station (same as user 2). Here, pricing is $\lambda = 10^{-4}$. Fig. 3 shows iterations 15 to 35 of our algorithm, where the new user enters at iteration 20. As shown in Fig. 3, when user 4 starts its transmission, it increases interference, i.e., reduces the actual SINR of other users. But our iterative algorithm converges again after some iterations to a point at which all users achieve their target SINRs. In addition, due to increased interference, all users (users 1, 2, and 3) increase their transmit power levels and decrease their data rates to compensate for the added interference and achieve their target SINRs.

In our next simulation, we examine base station assignment by considering two base stations and five users. All users are fixed except user 3 that moves 10 meters away from base station 1 towards base station 2 at each step. Users' distances from base stations 1 and 2 are $[110, 130, 210, 390, 410]$ and $[410, 390, 310, 130, 110]$, respectively at step 1, $[110, 130, 220, 390, 410]$ and $[410, 390, 300, 130, 110]$, respectively at step 2, and $[110, 130, 310, 390, 410]$ and $[410, 390, 210, 130, 110]$, respectively at the last step. The results are shown in Fig. 4, where we show the converged values at each step. From the first step to the sixth step, the base station to which user 3 is assigned is base station 1. Note that in step 6, user 3 is located at equal distances from the two base stations, i.e., 260 meters. However, when user 3 goes further towards base station 2 in step 7, our scheme allocates it to base station 2. In next steps, since user 3 goes near base station 2, its channel becomes better and hence it consumes less power and transmits at a higher data rate, as shown in Fig. 4.

We now compare our proposed scheme to [14] which is recently published and is very close to our work. We first provide results for two scenarios, where in the first one, users in the network are situated at different distances from the base station, and in the second, they have the same distance from the base station. The results are shown in Table II. In Scenario 1, the network consists of one base station and 5 users located at [110, 130, 210, 130, 150] meters from the base station. We set the packet length to 100 bits in [14], $\alpha_1 = 10^6$, $\alpha_2 = 12.9492$, and $\lambda = 0.0004$ for all users. The value of $\lambda = 0.0004$ results in achieving the target SINR which is set by the chosen values of α_1 and α_2. The value of α_2 for two

scenarios is set in such a way that our scheme achieves the same SINR as in [14]. Besides, $P_i^{\min} = 10^{-6}$ Watts, $P_i^{\max} = 0.1605$ Watts, $R_i^{\min} = 0.1$ bps, and $R_i^{\max} = 96000$ bps. The value of $P_i^{\max}$ is chosen so that the maximum achieved transmit power would be the same in both schemes. For Scenario 1, the total data rate of users in [14] is 110890 bps, while this value in our scheme is 99852 bps (slightly less than that in [14]), but the total transmit power in our scheme is 0.3914 watts, which is significantly less than 0.8025 watts in [14]. In other words, we consume nearly 50% less power at the cost of a slight reduction in total throughput. In addition, one can see that the user with the worst channel, i.e., user 3, attains a higher throughput (i.e., a better degree of fairness) in our scheme than in [14]. In Scenario 2, we consider 5 users, each situated 110 meters from the base station. The setting here is the same as in Scenario 1, except for $P_i^{\max} = 0.0486$ Watts. Note that in Table II for Scenario 2, the settings result in users' transmit power levels, users' data rates, and users' attained SINRs to be the same in both schemes.

Next, we compare our proposed scheme with [14] for different number of users. The results are shown in Table III. Each row in the table corresponds to one comparison. Note that we use the same settings as in Scenario 2, i.e., $P_i^{\max} = 0.0647$ Watts for both schemes, and transmit power, data rate, and achieved SINR are the same for all users. The first 5 rows in Table III correspond to 5 different cases, each with 3, 4, 5, 6, and 7 users all located at the same distance of 110 meters from the base station. As can be seen in Table III, throughput values of users in the first two rows are the same in both schemes. However, for the same value of the total throughput for our scheme and [14], power consumption in our scheme is less than $P_i^{\max}$, which is the transmit power in [14]. When the number of users in the network increases from 5 to 6 and 7, as the power level of users reaches its upper bound, the data rates are obtained from (28). Note that, although the throughput of users in our scheme is higher than that in [14], the achieved SINR values of users are less than their target values as the convergence point of the algorithm is out of the power strategy space. However, there exists one additional degree of freedom in our scheme, i.e., the pricing λ. In rows 6 and 7 of Table III, we again simulate the network with 6 and 7 users but with $\lambda = 0.0005$ and $\lambda = 0.0006$, respectively. Due to the higher pricing, users tend to transmit at less power and/or at less data rate. For these values for the pricing factor, the power level, the data rate, and the achieved SINR of users are the same in both scheme.

Finally, we compare the performance of our proposed scheme with that of [14] when the distances of users change. The network consists of 10 users and 1 base station. We set noise power to $N_0 = 10^{-10}$, $\lambda = 0.0001$, and $P_i^{\max} = 1$ Watts for all users. All other settings are the same as before except for the distance of users. Users are located at the same distance from the base station. The distance of users to base station at each run is 50, 150, 250, and 350 meters, respectively. The results are shown in Table

IV. Note that when users are located near to the base station, the data rates of all users are equal in both schemes. However, our proposed scheme consumes much less power than that of [14]. When users are located at 350 meters from the base station, our scheme reaches its upper bound on the power level. Similar to the previous cases in Table III, although the data rate of users are higher in our scheme, the achieved SINR is less, meaning that users did not achieve their target SINRs. However, we can increase the pricing so that all users achieve their target SINRs. This is shown in the last row of Table IV.

VII. Conclusions

In this paper, we proposed a distributed scheme for joint data rate and power control in CDMA networks. We formulated the problem by utilizing a game theoretic framework, proved the existence and uniqueness of NE for the game, and proved that our distributed algorithm always converges to this unique NE. Also, we showed that at NE, each user attains its target SINR, and that the proposed algorithm is computationally efficient and its signalling overhead is low. Simulation results validated our analysis.

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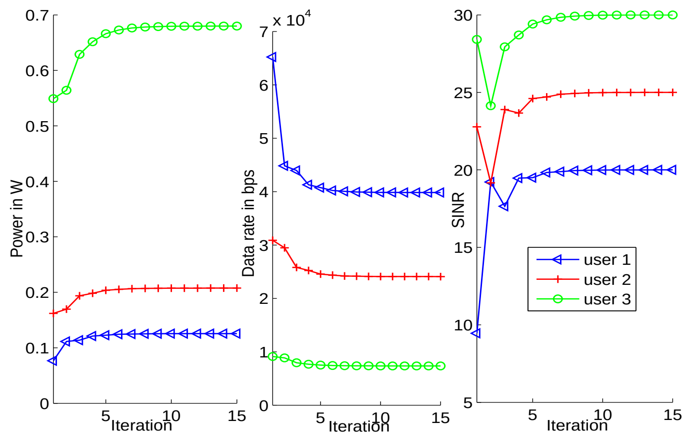

Fig. 1. Transmit power levels, data rates, and attained SINRs for our proposed resource allocation algorithm for different target SINRs. Note that every user attains its target SINR.

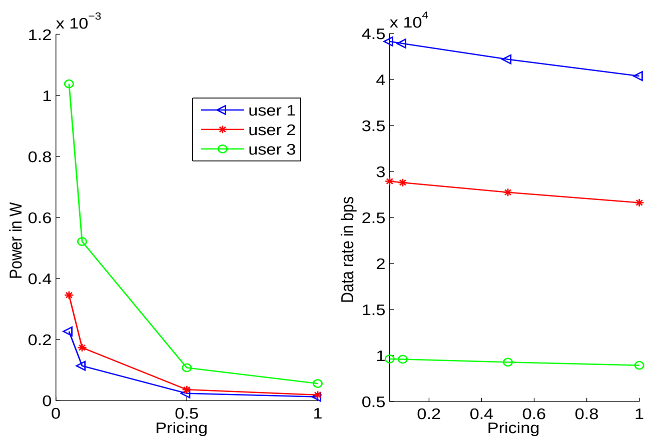

Fig. 2. The impact of increased pricing on the converged values of transmit power levels and data rates.

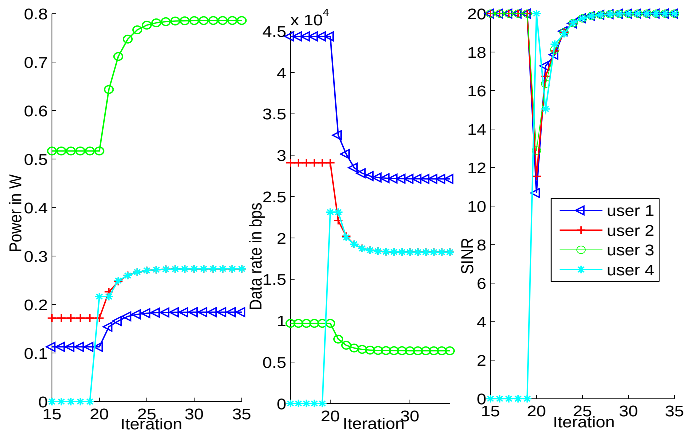

Fig. 3. The impact of a new user on transmit power levels, data rates, and attained SINR in our proposed resource allocation algorithm. Note that a new user only temporarily affects the attained SINR of existing users.

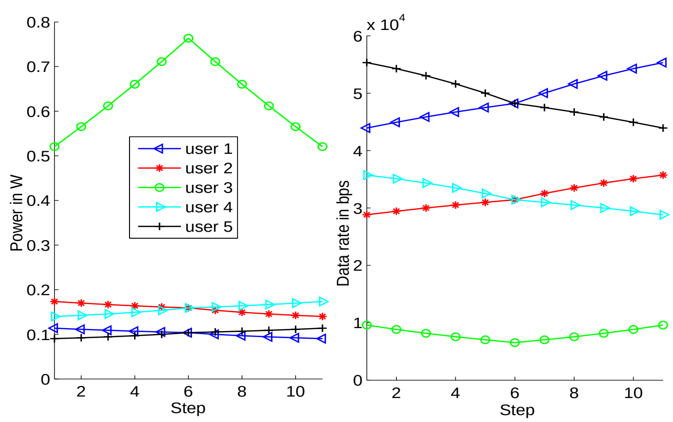

Fig. 4. Transmit power levels and data rates in our proposed resource allocation algorithm. Each step on the horizontal axis corresponds to a sufficient number of iterations needed for convergence.

TABLE I

AN EXAMPLE IN WHICH THE CONVERGENCE POINT IS OUT OF STRATEGY SPACE, WHERE THE EFFECTS OF INCREASED PRICING AND USER REMOVAL ON THE CONVERGENCE POINT ARE SHOWN

	$\lambda = 10^{-5}$			$\lambda = 10^{-4}$			User 3 is Removed		
	p_i in W	r_i in bps	Target SINR	p_i in W	r_i in bps	Target SINR	p_i in W	r_i in bps	Target SINR
User 1	1.011	47000	21.0452	0.1127	44360	20	0.08	47000	26.58
User 2	1.5533	32189	20	0.172	29075	20	0.125	40000	20
User 3	3	10205	12.2458	0.5166	9679	20	-	-	-

TABLE II

POWER LEVEL, DATA RATE, AND THE ACHIEVED SINR FOR EACH USER IN OUR PROPOSED SCHEME AND IN [14] WHEN USERS ARE LOCATED AT DIFFERENT DISTANCES (SCENARIO 1) AND AT EQUAL DISTANCE (SCENARIO 2) FROM THE BASE STATION

		Our Proposed Scheme			The Scheme in [14]		
		p_i in W	r_i in bps	Target SINR	p_i in W	r_i in bps	Target SINR
Scenario 1	User 1	0.0388	32201	12.9492	0.1605	55567	12.9492
	User 2	0.0569	21949	12.9492	0.1605	21089	12.9492
	User 3	0.1605	7787	12.9492	0.1605	2511	12.9492
	User 4	0.0569	21949	12.9492	0.1605	21089	12.9492
	User 5	0.0782	15982	12.9492	0.1605	10632	12.9492
Scenario 2	User 1	0.0647	19306	12.9492	0.0647	19306	12.9492
	User 2	0.0647	19306	12.9492	0.0647	19306	12.9492
	User 3	0.0647	19306	12.9492	0.0647	19306	12.9492
	User 4	0.0647	19306	12.9492	0.0647	19306	12.9492
	User 5	0.0647	19306	12.9492	0.0647	19306	12.9492

TABLE III

COMPARISON OF THE PERFORMANCES OF OUR PROPOSED SCHEME WITH THAT OF [14] FOR DIFFERENT NUMBER OF USERS AND PRICING λ

Number of Users	Pricing Factor	Our Proposed Scheme			The Scheme in [14]		
		p_i in W	r_i in bps	Target SINR	p_i in W	r_i in bps	Target SINR
M=3	$\lambda = 4 \times 10^{-4}$	0.0324	38612	12.9492	0.0647	38612	12.9492
M=4	$\lambda = 4 \times 10^{-4}$	0.0486	25741	12.9492	0.0647	25741	12.9492
M=5	$\lambda = 4 \times 10^{-4}$	0.0647	19306	12.9492	0.0647	19306	12.9492
M= 6	$\lambda = 4 \times 10^{-4}$	0.0647	17274	11.578	0.0647	15445	12.9492
M=7	$\lambda = 4 \times 10^{-4}$	0.0647	15769	10.569	0.0647	12871	12.9492
M= 6	$\lambda = 5 \times 10^{-4}$	0.0647	15445	12.9492	0.0647	15445	12.9492
M=7	$\lambda = 6 \times 10^{-4}$	0.0647	12871	12.9492	0.0647	12871	12.9492

TABLE IV

COMPARISON OF THE PERFORMANCES OF OUR PROPOSED SCHEME WITH THAT OF [14] FOR DIFFERENT DISTANCES OF USERS AND PRICING λ

User Distance	Pricing Factor	Our Proposed Scheme			The Scheme in [14]		
		p_i in W	r_i in bps	Target SINR	p_i in W	r_i in bps	Target SINR
$d = 50$ Meters	$\lambda = 1 \times 10^{-4}$	0.583	8570	12.9492	1	8570	12.9492
$d = 150$ Meters	$\lambda = 1 \times 10^{-4}$	0.635	8110	12.9492	1	8110	12.9492
$d = 250$ Meters	$\lambda = 1 \times 10^{-4}$	0.879	5686	12.9492	1	5686	12.9492
$d = 350$ Meters	$\lambda = 1 \times 10^{-4}$	1	3972	10.287	1	3155	12.9492
$d = 350$ Meters	$\lambda = 1.6 \times 10^{-4}$	1	3155	12.9492	1	3155	12.9492